The amazing story of a forgotten golden flag

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Abstract. We describe the most probable geometric design of the Chilean Independence Flag, which uses the golden ratio in many of its components. We also discuss some related historical aspects.

Several national flags incorporate mathematical elements in their design, which in most cases are fixed by law and sometimes are even explicitly stated in the corresponding national constitution. This is for instance the case of the flag of the United States of America, for which a nice and careful study about the symmetry rules for the arrangement of the stars is given in [3]. Most frequently, the involved elements are of a geometric nature, and are related to the proportions of the different components. One of the most remarkable example is that of Nepal's flag, the only non rectangular national flag [7], for which a quite intricate geometric construction [2] yields the width-height ratio

$$\frac{24 + \frac{297-180\sqrt{2}}{92-36\sqrt{2}} \left(1 + \frac{8-3\sqrt{2}}{\sqrt{118-48\sqrt{2}}-6}\right)}{32 + \frac{297-180\sqrt{2}}{92-36\sqrt{2}} \left(1 + \frac{6}{(8-3\sqrt{2})\left(\sqrt{1+\frac{18}{41-24\sqrt{2}}}-1\right)}\right)} \sim 0.820...$$

Another geometrically interesting flag is the Iranian one. Besides many beautiful elements in its design [8], the configuration forces a width-height ratio equal to

$$\frac{75}{28}(7\sqrt{5} - 15) \sim 1,747...$$

In many other cases, this ratio is in relation to the golden mean, either to its Fibonacci approximations (e.g. Palau, Poland and Sweden's flags [7]) or its genuine value (this is the case of Togo's flag [9]). It should be emphasized that this ratio is an irrational number only for three national flags: those of Nepal, Iran and Togo. The purpose of this Note is not at all to give a complete account of all mathematical aspects of the different national flags. Instead, we would like to draw the attention to the geometric beauty of another one, the Chilean Independence Flag, whose design is also based on golden proportions and remains widely unknown despite its two hundred years of history.

As it was the case of most independence movements in Latin America in the early 19th century, the Chilean one was guided by the new political and social ideas that inspired prior processes in both the United States of America and France. Besides, the Chilean process was led by a masonic intelligentsia (the so-called *Logia Lautaro*[1] that transferred its symbolism

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[1] *Lautaro* was, in the 16th century, the most important military leader (the *Cacique*) of the *Mapuche*, the largest indian community in the country that resisted the Spanish colonization. (His genuine name was *Leftraru*, and "Lautaro" is nothing but the Spanish version of it.)
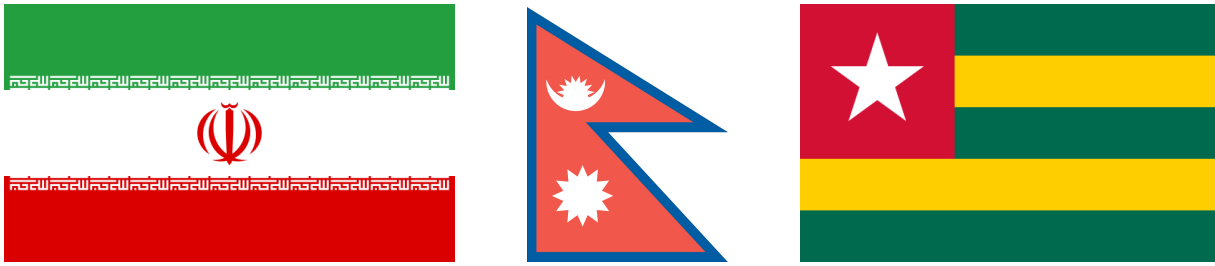
Figure 1: From left to right: the flags of Iran, Nepal, and Togo.

to many of the national emblems [1]. For instance, the colors used in the current Chilean flag are white, blue and red.

Although the original Independence Flag was quite similar to it, there are several remarkable differences. For example, following the explicit order of the military leader Bernardo O'Higgins, the Independence Flag included two shields (one with an obelisk and another one with a volcano), as well as an octogonal star (inside a pentagonal one) representing the planet Venus, as was usually depicted by the Mapuche. However, from a mathematical point of view, the most spectacular differences concern geometry, as explained below.

It is worth emphasizing that no official document concerning the design of the Independence Flag remains. Actually, it is not even clear that such a document ever existed. In particular, the author of this design is unknown, although most historians claim that it is Antonio Arcos, a Spanish military engineer who fought on the Chilean side during the Independence War. Besides, there is just one copy of the flag, which was indeed used during the Independence Signature Ceremony (February 12th, 1818) and is preserved in the National Historic Museum at Santiago.[2] Since then, only one person – the musician and philosopher Gastón Soublette [6] – noticed that the design incorporates the use of the golden mean. Unfortunately, his analysis was incorrect as it was guided mainly by symbolic elements and not by geometric ones.

What follows is a summary of the analysis in [5] leading to what most probably was the geometrical design of the Chilean Independence Flag. Notice that the description below is modeled on, yet not always coincides with, the measures of the existing flag, as this is somewhat deformed and has been restored in many ways, without paying much attention to its proportions; see [4].
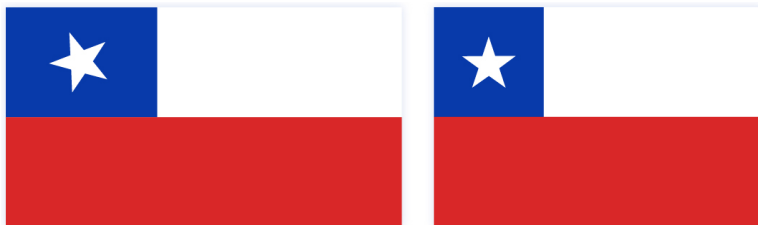

Figure 2: On the left, the (reconstructed) Independence Flag (without the shields and the octogonal star); on the right, the current flag.

Like the current flag, the Independence Flag was composed of three regions: a red rectangle at the bottom, and a blue and a white rectangle at the top (the white one being to

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[2]This has been so since the independence, except that in 1980 it was stolen as a sign of resistance to the Pinochet dictatorship. (It was belatedly returned in 2003.)

the right of the blue one), all of them with the same height. The ratio of the widths of the white and blue parts equals the golden mean. The blue part corresponds to a rectangle for which the proportion between the height and width equals

$$\tan(36^{\text{o}}) = \frac{\sqrt{10 - 2\sqrt{5}}}{1 + \sqrt{5}} = \frac{\sqrt[4]{5}}{\sqrt{2 + \sqrt{5}}} \sim 0.726...$$

This choice leads to the quite elegant angle configuration below, in which several isosceles triangles of angles $36^{\text{o}}, 72^{\text{o}}, 72^{\text{o}}$ appear, so that golden proportions are omnipresent.[3] Besides, this makes the width-height ratio of the flag to be equal to

$$\frac{2 + \sqrt{5}}{\sqrt{10 - 2\sqrt{5}}} \sim 1.801...$$

Finally, a white pentagonal star is depicted in the blue rectangle with its center in the intersection point of the diagonals, as shown below. The size of this star is very special: the ratio between the height of the rectangle and the diameter of the star's circumcircle equals, again, the golden mean.
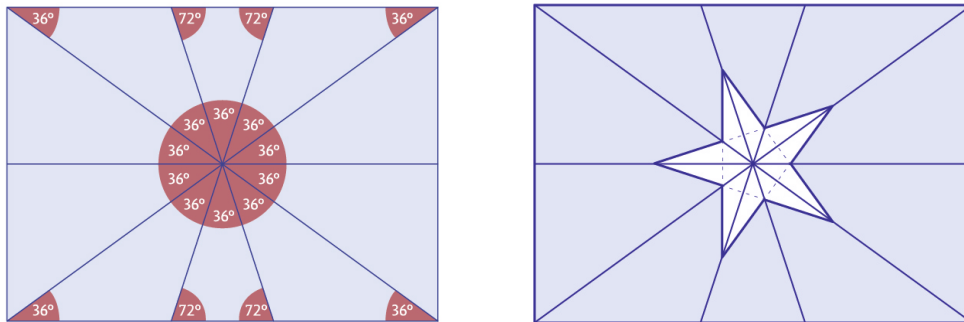

Figure 3: The angle configuration and the star.

With this configuration, the Independence Flag was elaborated with a 2.4 m width size. Concerning the current flag, it was already in use in the middle of the 19th century (yet the final law establishing its proportions dates back to 1912): the width-height proportion is $3:2$, so that it is composed of six squares (3 red squares, 2 white ones and a blue one); the star is centered at the center of the blue square, and the diameter of its circumcircle is half the length of the side of the square. Needless to say, this was certainly a regrettable simplification.

It is now the task of the historians to determine the reasons for this very quick replacement and the lack of major information about the original Independence Flag: was this due to the complexity of the design and the difficulties for the elaboration at that time, or was it the result of internal disputes between the independence leaders and the first rulers of the country?[4]

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[3]This is to be compared with the Chinese national flag, whose configuration also incorporates angles of 36°; see [10].

[4]Two of the three independence leaders, José Miguel Carrera and Manuel Rodríguez, were executed, most probably with the consent of the Logia Lautaro; the third one, O'Higgins, ruled the country up to 1823, when he was exiled to Perú, where he died in 1842.

Figure 4: On the left, a picture of the genuine Independence Flag; on the right, a view of the shield in the reverse side.

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[9] https://en.wikipedia.org/wiki/Flag_of_Togo

[10] https://en.wikipedia.org/wiki/Flag_of_China

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