Groups, Orders, and Dynamics
Second Lecture

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YGGT meeting
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References

- *Probabilistic and dynamical aspects of (left) orderable groups*.
  A. Navas; see
  http://imerl.fing.edu.uy/coloquio2/materiales/Curso_Navas.pdf

- *Orderable Groups*.
  R. Botto-Mura, A. Rhemtulla.

- *Right-ordered Groups*.
  V. Kopytov, V. Medvedev.

Different types of orders

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Theorem (Hölder)

Every Archimedean ordered group is ordered-isomorphic to a subgroup of $\mathbb{R}$.

Idea of Proof. Fix $f \succ id$ and define

$$g \longrightarrow \lim_{m\to\infty} \frac{n}{m}, \qquad f^{n-1} \preceq g^m \prec f^n$$

Locally indicable groups

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Theorem (Conrad; Brodskii, Rhemtulla-Rolfsen, N)

Local indicability is equivalent to Conrad-orderability:

$$f \succ id, g \succ id \implies fg^n \succ g \text{ for some } n \geq 1$$

A concrete application

Theorem (Witte, conjectured by Linnell, Thurston)

Every left-orderable amenable group is locally indicable.

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Question

What about left-orderable groups without free subgroups?

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$\mathcal{LO}(\Gamma)$: set of all left-orders of Γ

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$\mathcal{BO}(\Gamma)$: closed subspace (fixed points of this action).

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Theorem (Witte, conjectured by Linnell, Thurston)

Every left-orderable <u>amenable</u> group is locally indicable.

Proof:

- Consider the Γ-action on $\mathcal{LO}(\Gamma)$: $f_1 \prec_g f_2 \quad \text{iff} \quad f_1 g^{-1} \prec f_2 g^{-1}$
- Γ amenable $\implies$ it preserves a probability measure on $\mathcal{LO}(\Gamma)$

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(The set of orders for which f is positive must come back to itself under iterates of g^{-1}.)

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A μ-generic $\preceq$ is Conradian !

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Proof [Jiménez]. Assume $f \succ id$, $g \succ id$, but $fg^2 \prec g$.

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$$\begin{aligned} fh^n = f(fg)^n = f(fg)^{n-2}(fg)(fg) \prec f(fg)^{n-2}(fg)g \\ = f(fg)^{n-2}fg^2 \prec f(fg)^{n-2}g \\ = f(fg)^{n-3}fg^2 \prec f(fg)^{n-3}g \\ \vdots \\ \prec f(fg)g = ffg^2 \prec fg = h. \end{aligned}$$

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- $X =$ bi: *id* does not belong to the smallest normal subsemigroup containing $g_i^{\epsilon_i}$.
- X= Conrad: *id* does not belong to the smallest semigroup containing $g_i^{\epsilon_i}$ and that contains all elements of the form $g^{-1}fg^2$ for all f, g therein.

Conrad-orderability of locally-indicable groups (Brodskii)

Fix nontrivial elements $g_1, \ldots, g_k$ in Γ, and denote by S the semigroup above.

- Take a nontrivial $\phi_1 \colon \langle g_1, \ldots, g_k \rangle \to (\mathbb{R}, +)$.
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- For every f, g in S for which a certain ϕ_j is defined, one has

$$\phi_j(f^{-1} g f^2) = \phi_j(f) + \phi_j(g)$$

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  Moreover, it is "uniformly" positive if for either f or g is positive.

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Theorem (N)

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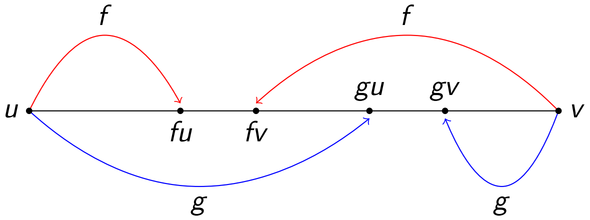

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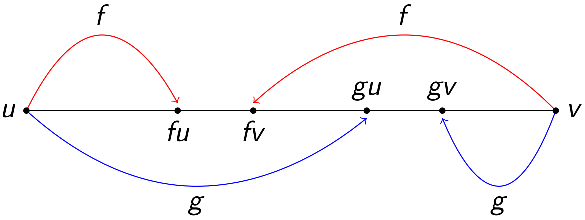

The lack of the Conrad property generates "interesting dynamics"